

# Q-Ball Collisions in the MSSM

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## Abstract

Collisions of non-topological solitons, Q-balls, are studied in the Minimal Supersymmetric Standard Model in two different cases: where supersymmetry has been broken by a gravitationally coupled hidden sector and by a gauge mediated mechanism at a lower energy scale. Q-ball collisions are studied numerically on a two dimensional lattice for a range of Q-ball charges. Total cross-sections as well as cross-sections for fusion and charge exchange are calculated.

## 1 Introduction

A scalar field theory with a spontaneously broken  $U(1)$ -symmetry may contain stable non-topological solitons[1, 2], Q-balls. A Q-ball is a coherent state of a complex scalar field that carries a global  $U(1)$  charge. In the sector of fixed charge the Q-ball solution is the ground state so that its stability and existence are due to the conservation of the  $U(1)$  charge. In realistic theories Q-balls are generally allowed in supersymmetric generalizations of the standard model with flat directions in their scalar potentials. Q-balls have been shown to be present in the MSSM[3, 4] where leptonic and baryonic balls may exist and they may be formed in the early universe by a mechanism that is closely related to the Affleck-Dine baryogenesis[4, 5, 6, 7].

Stable Q-balls can contribute to the dark matter content of the universe. These can be balls with charges of the order of  $10^{20}$  but also very small Q-balls can be considered as dark matter[5, 8]. On the other hand decaying Q-balls can protect baryons from the erasure of baryon number due to sphaleron transitions by decaying after the electroweak phase transition[4]. Q-ball decay may also result in the production of dark matter in the form of the lightest supersymmetric particle (LSPs). This process may explain the baryon to dark matter ratio of the universe[9].

## 2 Q-ball solutions

Consider a field theory with a  $U(1)$  symmetric scalar potential  $U(\phi)$  that has a global minimum at  $\phi = 0$  and the complex scalar field  $\phi$  carries a unit charge with

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respect to the  $U(1)$ -symmetry. The charge and energy of a field configuration  $\phi$  are given by

$$Q = \frac{1}{i} \int (\phi^* \partial_t \phi - \phi \partial_t \phi^*) d^3x \quad (1)$$

$$E = \int d^3x [|\dot{\phi}|^2 + |\nabla \phi|^2 + U(\phi)]. \quad (2)$$

The Q-ball solution is the minimum energy configuration at a fixed charge. If it is energetically favourable to store charge in a Q-ball compared to free particles, the Q-ball will be stable. Hence for a stable Q-ball, condition  $E < mQ$ , where  $m$  is the mass of the  $\phi$ -scalar, must hold.

The Q-ball solution can be shown to be of the form  $\phi(x, t) = e^{i\omega t} \phi(r)$ , where  $\phi(r)$  is now time independent, spherically symmetric and real.  $\omega$  is the so called Q-ball frequency,  $\omega \in [-m, m]$ . Q-balls can be characterized by the value of  $\omega$ : the larger  $\omega$  the larger the charge carried by the Q-ball.

### 3 Q-ball profiles

Q-balls and their cosmological significance have been studied in the MSSM mainly in two different types of potentials that correspond to SUSY broken by a gravity- or gauge-mediated mechanism.

The potentials in these two cases are respectively

$$U_{Gr}(\phi) = m_1^2 \phi^2 (1 - K \log(\frac{\phi^2}{M^2})) + \lambda_1 \phi^{10} \quad (3)$$

$$U_{Ga}(\phi) = m_2^4 (1 + \log(\frac{\phi^2}{m_2^2})) + \frac{\lambda_2^2}{m_{Pl}^2} \phi^6. \quad (4)$$

The parameter values we have chosen are:  $m_1 = 10^2$  GeV,  $K = 0.1$ ,  $\lambda_1 = m_{Pl}^{-6}$ ,  $m_2 = 10^4$  GeV,  $\lambda_2 = 0.5$ . The large mass scale,  $M$ , is chosen such that the minimum is degenerate.

Q-ball profiles are of different type in these two cases. In the gravity-mediated case the profiles are well approximated by a Gaussian ansatz. The radius of a Q-ball is only weakly dependent on charge and Q-balls are typically thick-walled. In the gauge-mediated case the profile of a ball is more dependent on charge and as charge increases the Q-ball profiles become thin-walled.

Q-ball profiles in the two cases have been plotted in figure 1 in two and three dimensions for different values of  $\omega$ .

### 4 Collisions

Collisions of Q-balls may play an important role in their cosmological history. After the Q-balls have been formed, collisions can alter the charge distribution significantly which in turn may have an effect on the importance of Q-balls to cosmology, *e.g.* if Q-balls fragment due to collisions they may evaporate[10] and cannot be responsible for the dark matter content of the universe. Q-ball collisions have been previously considered analytically[5] and numerically[11, 12, 13, 14] in various potentials. To address the cosmological issues, however, one needs to calculate cross sections in realistic potentials.

We have studied Q-ball collisions in the gravity- and gauge-mediated cases[15, 16] on a  $(2+1)$ -dimensional lattice for a range of Q-ball sizes (charges). As from

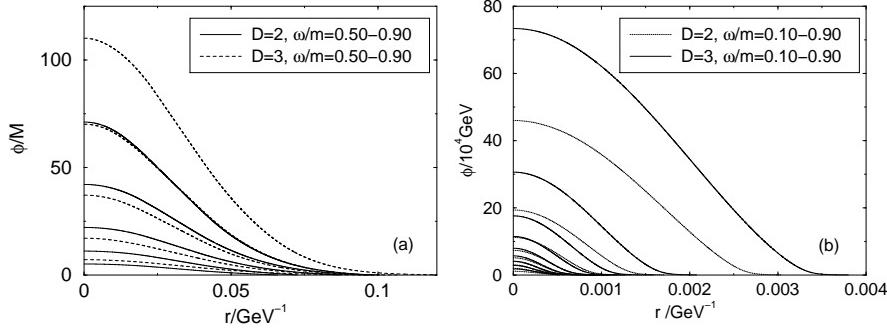


Figure 1: Q-ball profiles in the (a) gravity- and (b) gauge-mediated case in two and three dimensions

figure 1 can be seen, the Q-ball profiles in two and three dimensions are similar and one can expect that the results obtained in these two dimensional simulations well approximate the results from the more realistic three dimensional calculations.

Collisions have been studied for a range of charges, relative phase differences,  $\Delta\phi$ , and different velocities. In each case the colliding balls have equal charges. Collisions in both potentials have similar features: when the relative phase difference is small the balls fuse. In a fusion process some charge is typically lost as small lumps and radiation. As  $\Delta\phi$  increases the balls start to scatter while charge is transferred from one ball to the other. As  $\Delta\phi$  grows further the amount of transferred charge decreases until the Q-balls scatter elastically.

By varying the impact parameter the cross-sections for each process and the total cross-section have been calculated. The total,  $\sigma_{tot}$ , geometric,  $\sigma_G$ , fusion,  $\sigma_F$ , and charge-exchange,  $\sigma_Q$ , cross-sections averaged over the relative phase have been plotted in figures 2 and 3 for different  $\omega$ :s in the gravity- and gauge-mediated scenarios.

## 5 Conclusions

Q-ball collisions have been studied in the MSSM with supersymmetry broken by two different mechanisms. Even though the Q-balls in the two cases have differing characteristics, the qualitative features of a collision process are alike: Q-balls either fuse, exchange charge or scatter elastically. In both cases the relative phase difference between the colliding Q-balls determines the type of the collision.

## Acknowledgments

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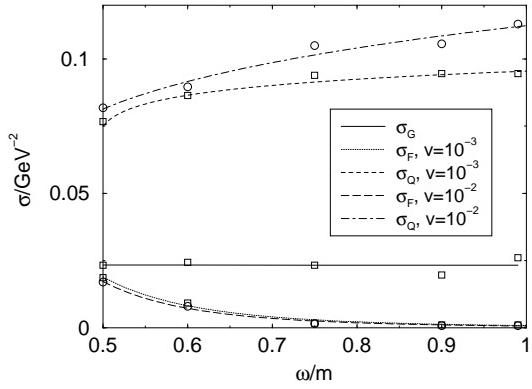


Figure 2: Geometric, fusion and charge-exchange cross-sections for  $v = 10^{-3}$  and  $v = 10^{-2}$  in the gravity-mediated scenario

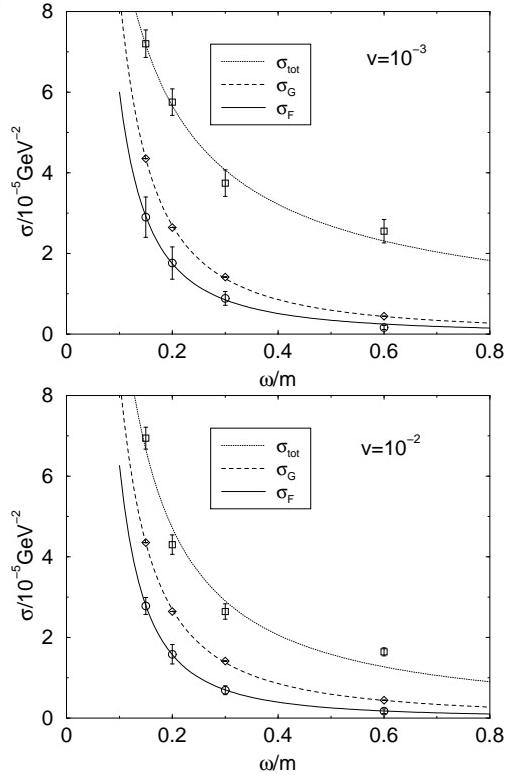


Figure 3: Total, geometric and fusion cross-sections for  $v = 10^{-3}$  and  $v = 10^{-2}$  in the gauge-mediated scenario